For example, in the Bertrand pricing game with two players, the strategy space for player i is $(0, \infty)$, and the payoff if player 1 plays $p_1 \in (0, \infty)$ is $p_1 d_1(p_1, p_2)$, where

$$d_1(p_1, p_2) = \begin{cases} d(p_1) & \text{if } p_1 < p_2 \\ d(p_1)/2 & \text{if } p_1 = p_2 \\ 0 & \text{if } p_1 > p_2 \end{cases} \quad \textbf{(F.1)}$$

and $d(\cdot)$ is the market-demand function.

Simultaneous Move Static Games

In the simultaneous move static game, all players move exactly once and make their moves simultaneously. Hence, no player knows what the other players' moves are going to be, nor do they have any information on past moves of their opponents (as it is a one-move game).

These are rather restrictive assumptions. Nevertheless, such games are applicable in some situations (for instance, a sealed-bid auction), and they serve as the basis for the study of more complicated repeated games.

Game theory is concerned with predicting the outcomes of a game assuming the players are rational (utility-maximizing players). To this end, we define the concept of *equilibrium,* essentially a prediction of the possible outcomes of the game. There are many equilibrium concepts, depending on the nature of the information, and the assumptions on players' behavior.

We assume that players have *complete information* about the game. Each player knows the strategy sets, utility functions, and any other relevant parameters for all other players, and they also know that all the other players are rational and, like themselves, have complete information.

Dominant Strategies

A strategy $s_i \in S_i$ is a *dominant strategy* for player i if his payoff from playing s_i is no less than that from playing any other of his strategies, for all possible strategies of the other players.

Formally, let the the vector $\mathbf{s}_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_N)$ represent strategies of all players other than i, and S_{-i}, the set of all vectors of all possible strategies of all the players other than i.

Then, for a game $\Gamma = [N, \{S_i\}, \{u_i(\cdot)\}]$, $s_i \in S_i$ is a dominant strategy if for all $s_i' \neq s_i$,

$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}), \quad \forall s_{-i} \in S_{-i}.$$

If rational player i has a strictly dominant strategy, it is reasonable to predict he would always play that strategy. There are very few games, however, where such dominant strategies exist.

Nash Equilibrium

Perhaps the most important and widely accepted notion on the outcome of games with rational players is the Nash equilibrium.

Nash equilibrium, definition A strategy vector $\mathbf{s}$ is a *(pure strategy) Nash equilibrium* for the game $\Gamma = [N, \{S_i\}, \{u_i(\cdot)\}]$ if for every player $i = 1, \ldots, N$, given the strategies of the other players $\mathbf{s}_{-i}$, his strategy s_i is optimal, that is,

$$u_i(s_i, \mathbf{s}_{-i}) \geq u_i(s_i', \mathbf{s}_{-i}), \quad \forall s_i' \in S_i, \ i = 1, \ldots, N.$$